Assumptions of the Kinetic theory of Gases:

Several idealised assumptions are made about the behaviour of ideal gas.

(1) A small sample of gas consists of a large number of molecules which are identical for a given chemical species and are like minute hard elastic spheres which are in perpetual random motion.

(2) During the motion, the molecules collide which one another and also with the walls of the container, the collision being perfectly elastic. in other words, there is no loss of kinetic energy when collisions occur.

(3) The molecules are inert and exert no forces on one another except when they actually collide with each other and with a wall; between two successive collisions they move in straight lines with uniform speed.

(4) The collisions are essentially instantaneous, that is, the duration of a collision is insignificant compared to the time betn two collisions.

(5) Since the molecules are like geometrical mass-points the actual volume occupied by them is negligible, compared to the total volume of the gas.

(6) There is no preferred direction for the velocity of any molecule so that at any ~~molecu~~ moment as many molecules move in one direction as in another. Not all molecules have the same speed. Few move slowly few very rapidly, ranging from 0 to the speed of light.

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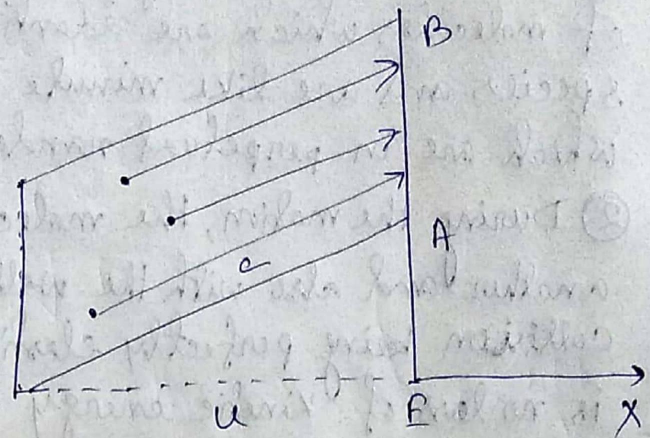
Deduction of ideal gas equation:

Consider an ideal gas enclosed in a container such that one of its surface, BAE in fig. is perpendicular to the X-axis.



Now, any velocity c of a particle in space can be resolved into three components u, v and w along the three axes so that

$$c^2 = u^2 + v^2 + w^2$$

the limit of u, v and w are from $-\infty$ to $+\infty$, those of c are from 0 to ∞

Now, u suffers a change in direction on reflection from BAE, the other two components v and w remain unchanged by this type of reflections.

The change in momentum per reflection of a molecule is $\quad m u - (-m u) = 2mu$

If n_u be the number of molecules per unit volume moving with velocity u, the number striking unit area of the wall in time dt would be contained in a cylinder of cross-sectional area unity and a vertical height $u\,dt$.

$\therefore$ vol^m of the cylinder $= u\,dt$

& Number of molecules in the cylinder $= n_u u\,dt$

∴ Change in momentum/area suffered by the above molecules in time dt is

$$2mu \times n_u u\,dt$$

The total change in momentum per unit area is

$$2m\sum_{0}^{\infty} n_u u^2 dt$$

If the above change in momentum results in an average force δF, then

$$\delta\vec{F} = 2m\sum_{0}^{\infty} n_u u^2$$

Since the area involved is unity, $\delta\vec{F}$ is also the pressure P.

$$P = 2m\sum_{0}^{\infty} n_u u^2$$

Let $\overline{u^2}$ be the mean square velocity along x,

$$\overline{u^2} = \frac{n_1 u_1^2 + n_2 u_2^2 + \cdots + n_m u_m^2 + \cdots}{n_1 + n_2 + n_3 + \cdots + n_m + \cdots}$$

$$= \frac{\sum n_i u_i^2}{\sum n_i}$$

$$= \frac{\sum n_i u_i^2}{n/2}$$

The factor $\frac{1}{2}$ arises due to the fact that only the molecules in +ve x direction are being considered.

$$\therefore \sum_{0}^{\infty} n_u u^2 = \frac{1}{2} n \overline{u^2}$$

$$\therefore P_x = 2m \times \frac{1}{2} n\overline{u^2} = mn\overline{u^2}$$

Similarly, $P_y = mn\overline{v^2}$

& $P_z = mn\overline{w^2}$

The expression for the pressure is

$$P = P_x = P_y = P_z = mn\overline{u^2} = mn\overline{v^2} = mn\overline{w^2}$$

But $\overline{u^2} = \overline{v^2} = \overline{w^2} = \frac{1}{3}\overline{c^2}$ here, $\overline{c^2}$ is the mean square velocity of molecule

$$\therefore \boxed{P = \frac{1}{3}mn\overline{c^2} = \frac{1}{3}mnC^2}$$

$C = \sqrt{\overline{c^2}}$ = Root mean square

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The expression for the pressure also be written as

$$P = \frac{1}{3}\rho c^2$$ here, $\rho = mn$
$= $ density of gas.

Root mean square velocity:

$$P = \frac{1}{3}\rho c^2$$

$$\therefore c = \sqrt{\frac{3P}{\rho}}$$

Consider one mole of the gas. then $\rho = \frac{M}{V}$
here, M is the molecular weight & V is volm.

$$\therefore P = \frac{1}{3}\frac{M}{V}c^2$$

$$\Rightarrow PV = \frac{Mc^2}{3} = RT$$

$$\therefore \boxed{c = \sqrt{\frac{3RT}{M}}}$$

Pressure and kinetic energy:

$$P = \frac{1}{3}\rho c^2$$

$$= \frac{2}{3} \times \frac{1}{2}\rho c^2$$

$$\boxed{P = \frac{2}{3}E_m}$$

here, $E_m = \frac{1}{2}\rho c^2$
$=$ mean kinetic energy of gas molecules per unit volm.

again, $E_m = \frac{1}{2}mc^2$

$$= \frac{1}{2}m \cdot \frac{3KT}{m}$$

$$\boxed{E_m = \frac{3}{2}KT}$$